ONR Uncertainty DRI Review Meeting June 2003

Analytic Bayesian and Adaptive Sonar Performance Prediction in an Uncertain Ocean

Liewei Sha, L.W. Nolte, Paul Book, and Jeff Krolik

Duke University
Department of Electrical and Computer Engineering
Durham, NC 27708, USA

Sponsored by ONR Code 321

Duke University
School of Engineering

Sonar Detection in an Uncertain Environment

OBJECTIVE: To achieve rapid characterization of sonar detection performance when *both* the ocean environment and the noise field directionality are uncertain.

BACKGROUND:

- When both the signal wavefront and noise covariance matrix are **known** *a priori,* the classical sonar equation bounds detection performance based on analytic PDF's.
- When both the ocean environment and noise covariance are **uncertain**, detection statistics used to predict performance in a known environment are not appropriate.
- Monte Carlo methods can be used to predict detection performance by randomizing over ocean parameters and noise covariance uncertainty but this is computationally intensive and gives little insight into the cause of performance degradation.
- In this project, recent analytic results for the performance of Bayesian and adaptive CFAR detectors have been applied to achieve rapid sonar performance prediction in an uncertain ocean environment.

Detection Performance Characterization

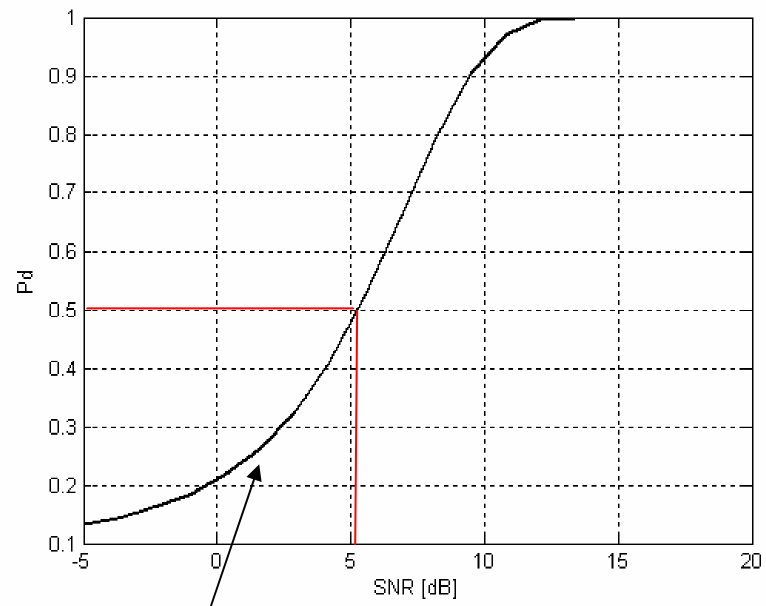
- Detection performance is characterized by the receiver operating characteristic (ROC) as a function of output SNR, time-bandwidth product, noise training data, and degree of signal wavefront uncertainty.

- The sonar equation summarizes the ROC as the output SNR (i.e. detection threshold, DT) needed to achieve a specific probability of detection (PD) and false alarm (PFA).

- A more complete description is given by PD vs. SNR for fixed PFA which, in principle, can be mapped to PD vs. range and bearing.

- Accurate performance prediction starts with using the PD vs. SNR curves appropriate when the ocean environment is uncertain.

Example PD vs. SNR for PFA=0.1



What are the right curves when the signal wavefront and noise covariance are uncertain?

Duke University School of Engineering

Adaptive CFAR Detection in an Uncertain Ocean

- In an uncertain waveguide, the *M*-sensor passive detection problem is given by:

$$\mathrm{H_o}: \; x_n = \eta_n \quad \text{versus} \quad \mathrm{H_1}: \; x_n = s_n U(\theta_s) a + \eta_n$$

where $[U(\theta_s)]_{ml} = \phi_l(z_m) e^{-jk_l md \sin\gamma \sin\theta_s}, [a(r_s, z_s)]_l = \phi_l(z_s) e^{-jk_l r_s}$, s_n are unknown amplitudes, θ_s, r_s, z_s are source bearing, range, depth, and γ is array tilt.

- We use $E(Uaa^H U^H) = H\Lambda H^H$ to define a reduced-dimensional signal subspace whose rank *p* increases with environmental uncertainty.

- Adaptive detection assumes a set of i.i.d "training vectors" are available to estimate the unknown Gaussian noise covariance, $R_\eta = E(\eta_n \eta_n^H)$

- For adaptive detection in an uncertain ocean, the CFAR generalized likelihood ratio test (GLRT) is given by:

$$\lambda(x_n) = \frac{x_n^H \hat{R}_\eta^{-1} H (H^H \hat{R}_\eta^{-1} H)^{-1} H^H \hat{R}_\eta^{-1} x_n}{K + x_n^H \hat{R}_\eta^{-1} x_n}$$

where $\hat{R}_\eta = \frac{1}{K} \sum_1^K x_n x_n^H$ estimated from K "signal-free" training data snapshots.

Duke University
School of Engineering

Analytic Probability of Detection vs. Output SNR

- If the signal wavefront and noise covariance are known exactly, the likelihood ratio test (LRT) detection statistic is non-central Chi-square distributed with PD:

$$PD(\tau) = \int_{\tau}^{\infty} \chi^2_{[2,2\cdot SNR]}(x)\mathrm{d}x$$

- Setting the level τ to achieve a specified PFA, this equation can be numerically solved to find the $SNR = |s|^2 a^H U^H R_\eta^{-1} U a$ associated with a specified PD.

- For an uncertain signal wavefront and noise covariance matrix, the statistics of the optimal adaptive CFAR GLRT have been derived by (Kraut et.al. 2001) with:

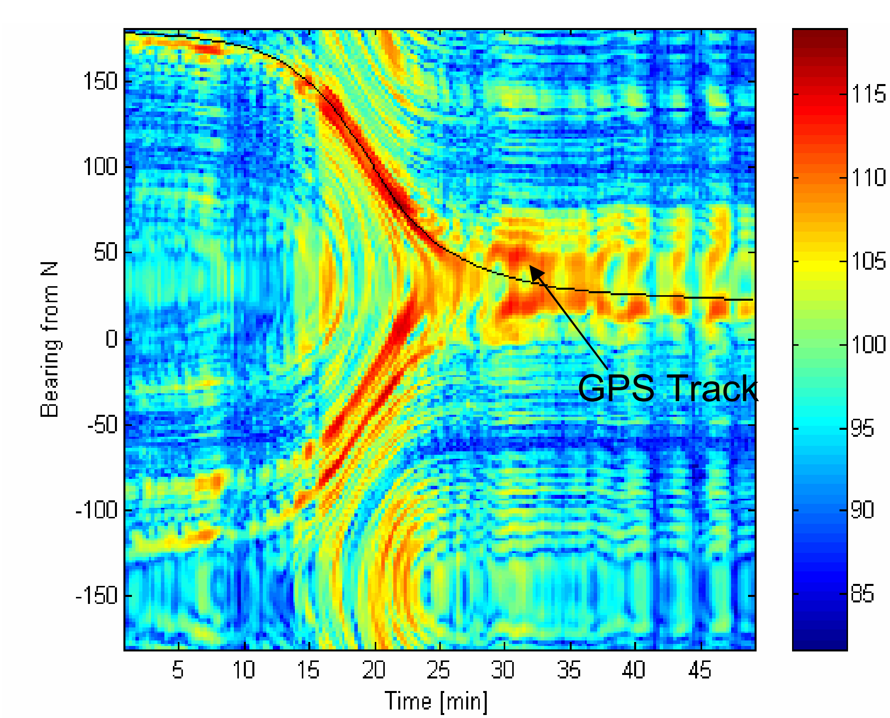
$$PD(\tau) = \int_{\tau}^{\infty}\int_{-\infty}^{\infty} F_{[2\cdot p, 2\cdot(K-M+1), 2\cdot SNR\cdot b]}(x)\mathrm{p}(b)\mathrm{d}b\mathrm{d}x \text{ with } \mathrm{p}(b) \sim \beta_{[K-M+p+1, M-p]}$$

- Now PD also depends on the ocean uncertainty through the signal subspace dimension, p, the number of training snapshots available to estimate the noise field, K, and the number of sensors in the array, M.

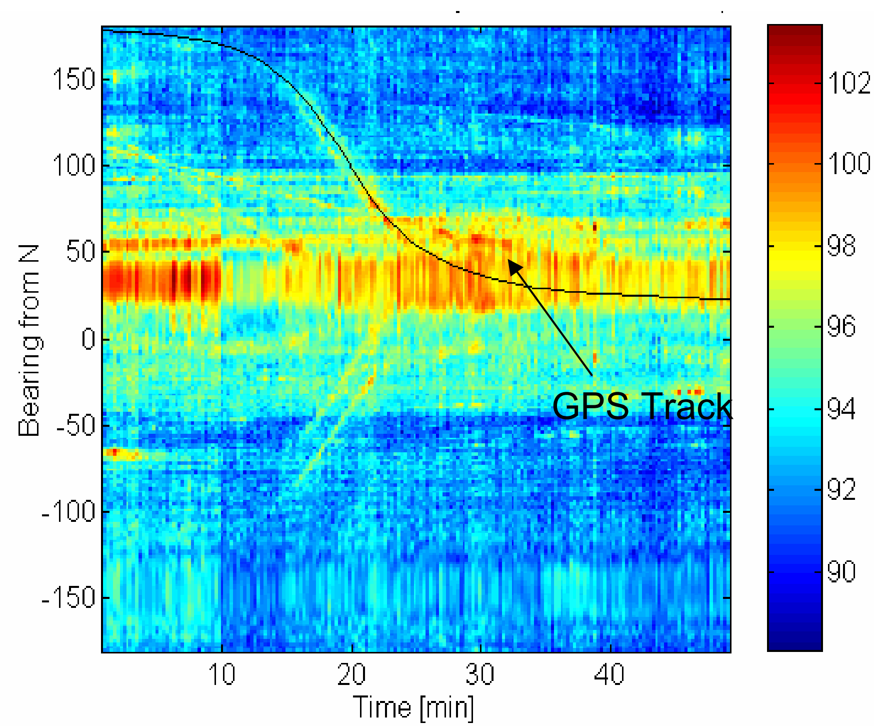
Horizontal Array Data from SWELLEX-96 Event S5

- The SWELLEX-96 dataset (courtesy MPL/SIO) was used to validate analytic PDF's and detection performance prediction.
- Narrowband data from the *M=27* sensor, 240-meter long, non-uniformly spaced horizontal line array North was compared with detection performance predictions for the S5 event. Bearing-time record for 109 Hz signal (right) and noise frequencies (left).

Source Ship Track Tonal at 109 Hz



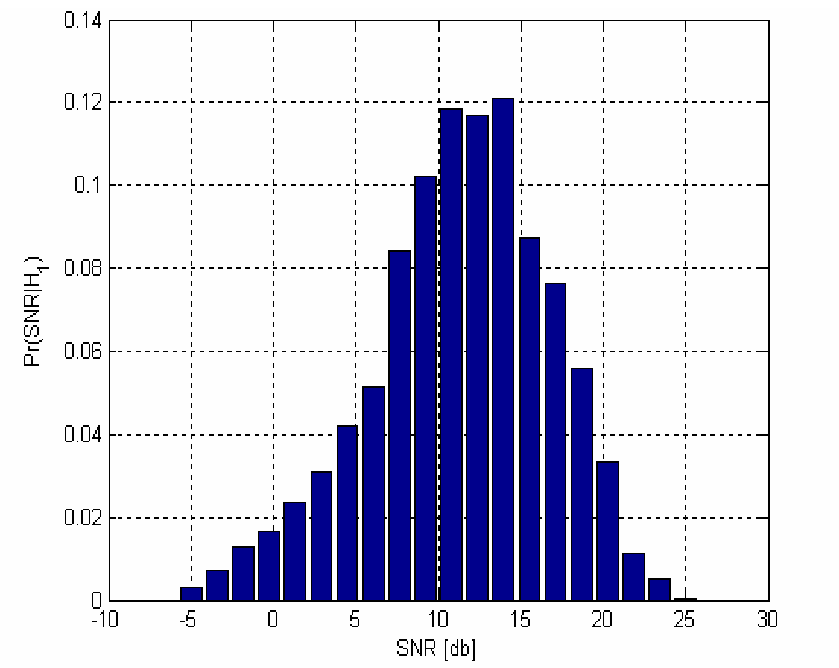
Noise Frequencies BTR



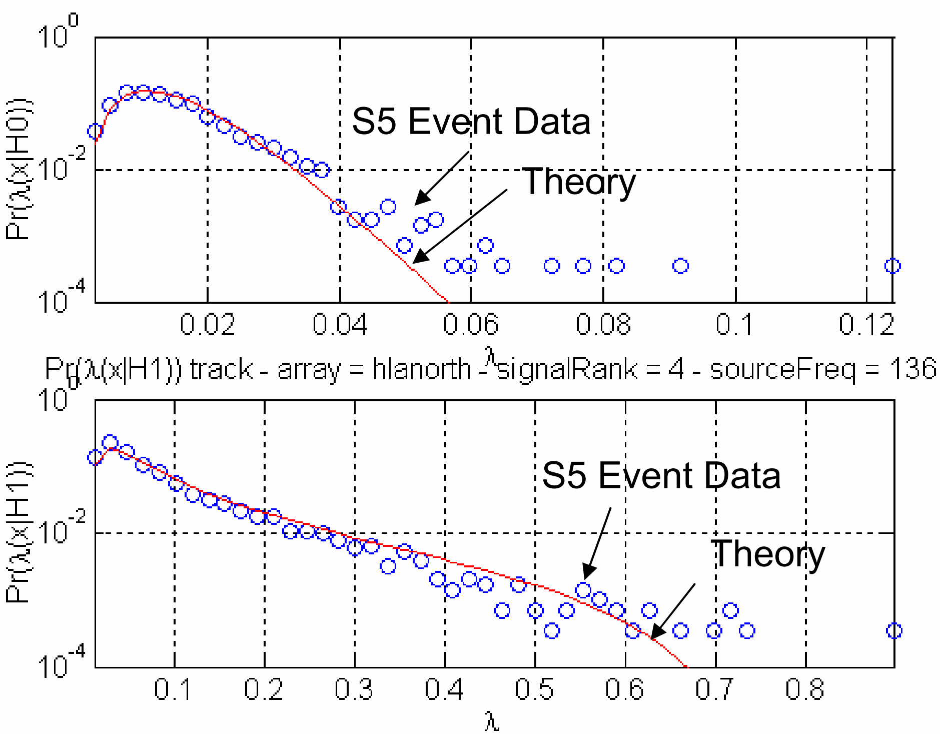
Detection Statistics Over Entire S5 Event Track

- Comparison of analytic PDF under H0 (noise only) and H1 (signal+noise) versus SWELLEX-96 data at 136 Hz for *p=4* and *K=300* (right figures) using estimated distribution of SNR's (left figure) over entire 50 minute track.
- Note good agreement between observed and predicted PDF's for this level of signal wavefront uncertainty.

Estimated PDF of S5 SNR over Track at 136 Hz



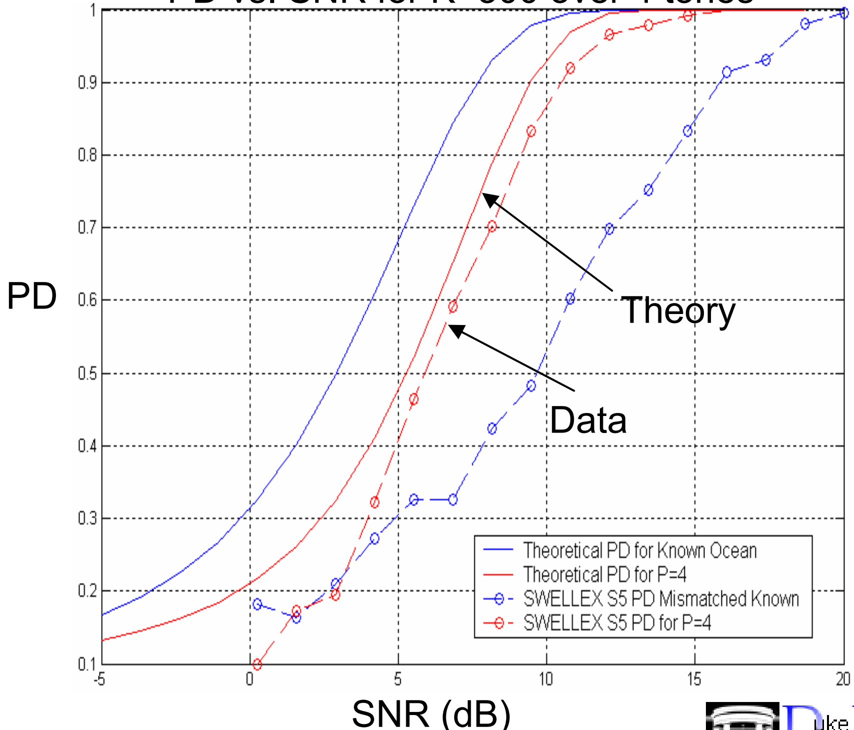
PDF of Detection Statistics vs. Theory over Track



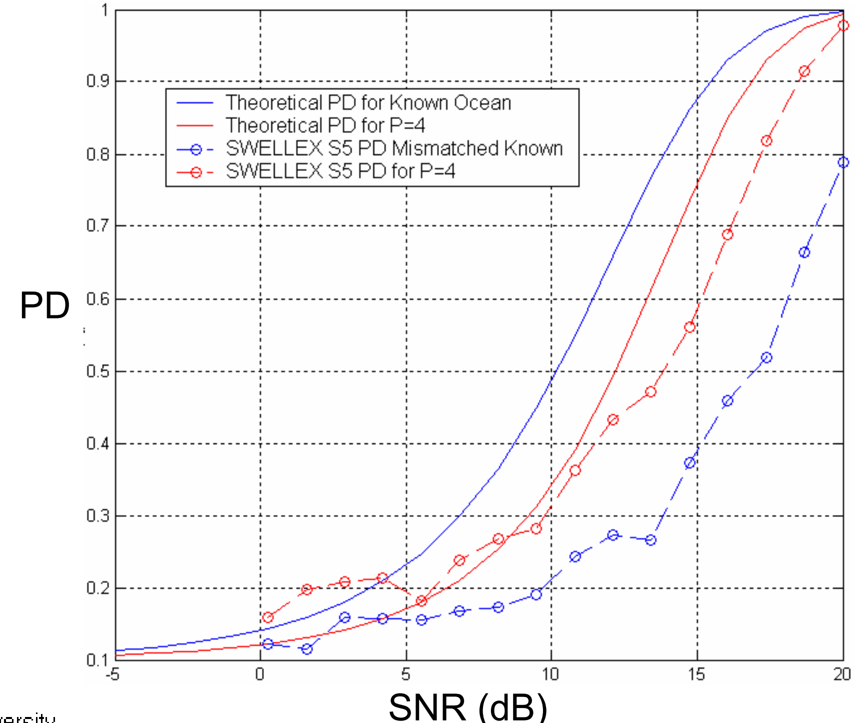
Event S5 Detection with Signal Wavefront Uncertainty

- Probability of detection versus SNR for the adaptive GLRT detector for real S5 M=27 HLA North event data versus analytic prediction with increasing signal wavefront uncertainty (i.e. subspace rank p). Snapshot support *K=300* (left) and *K=32* (right).
- Good agreement achieved between theory and data for uncertain wavefront *(p=4)* model (red curves). Note mismatch for *p=1* prediction versus real data when plane-wave beamforming assumed which may represent prediction error of current practice.

PD vs. SNR for K=300 over 4 tones



PD vs. SNR for K=32 over 4 tones



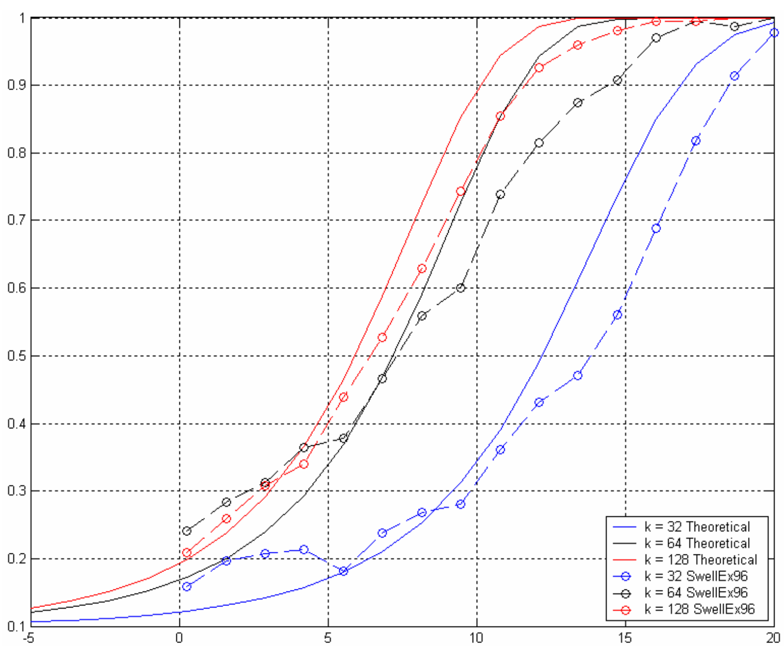
Duke University School of Engineering

Event S5 Detection with Limited Noise Training Data

- Probability of detection versus SNR for the adaptive GLRT detector for real S5 M=27 HLA North event data versus analytic prediction with decreasing training data (i.e. snapshots K). Signal wavefront uncertaint: *p=4* (left) and plane-wave *p=1* (right).
- Good agreement achieved between theory and data for uncertain wavefront (*p=4)* model (left) with mismatch evident when *p*=1 plane-wave modeling assumed (right). Note signal contamination in noise training data results in PD reduction at high SNR's.

PD vs. SNR for p=4 over 4 tones

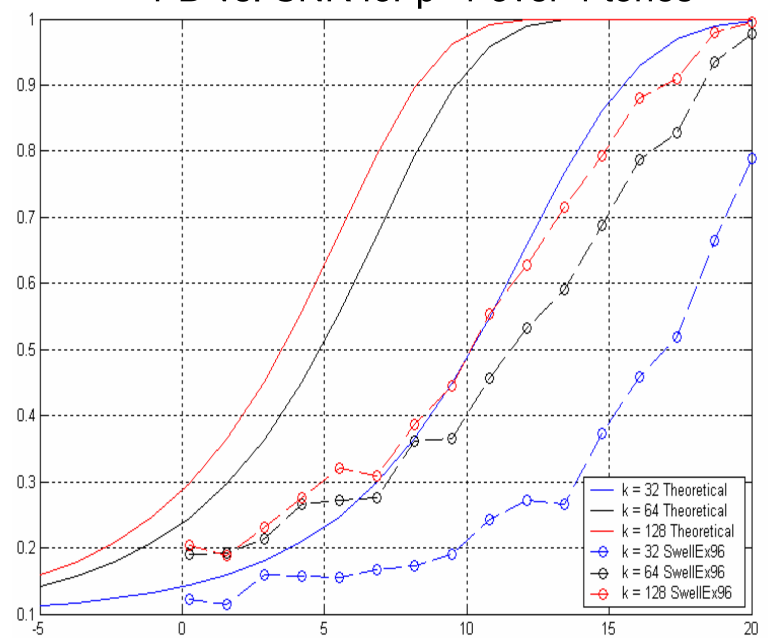
PD



SNR (dB)

PD vs. SNR for p=1 over 4 tones

PD



SNR (dB)

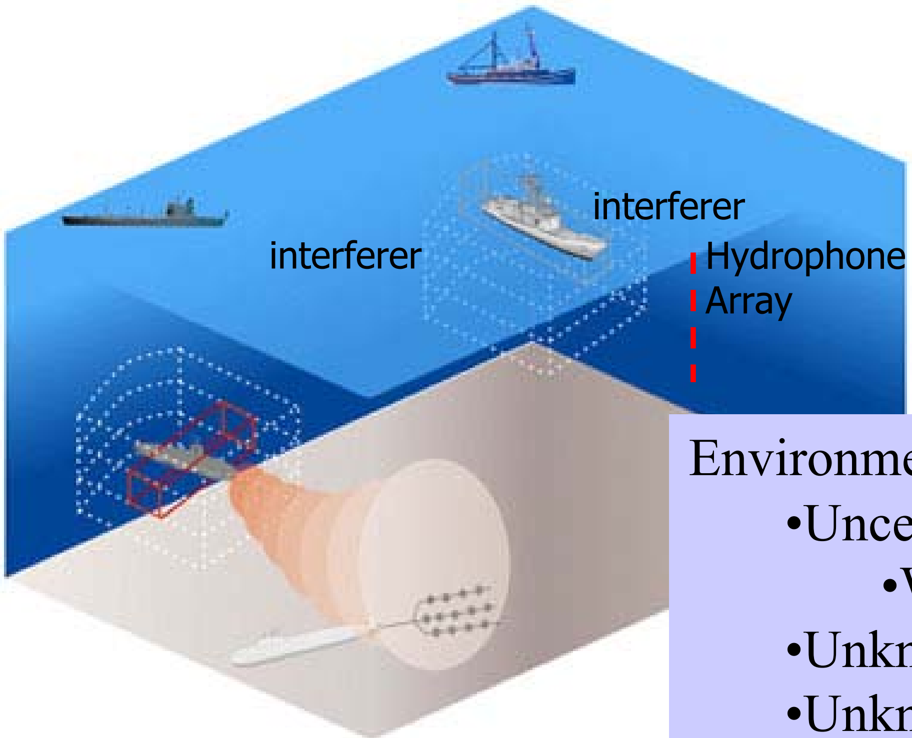
Duke University School of Engineering

Summary and Future Work

- When the signal wavefront and noise field is uncertain, analytic PDF's for the optimum adaptive matched-subspace detector (MSD) can be rapidly computed as a function of SNR, signal wavefront rank, and number of training data snapshots.
- The analytic PDF of the adaptive MSD agrees with S5 HLA North data when an uncertain wavefront is assumed and the number of noise snapshots is large.
- Measured PD vs. SNR agrees with analytic predictions when the signal wavefront is assumed uncertain and the number of noise snapshots is large.
- Measured PD vs. SNR agrees with analytic predictions at moderate SNR's when the signal wavefront is assumed uncertain for different number of training snapshots.
- At higher SNR's, signal contamination of the noise training data can cause the measured PD to be notably less than analytical prediction when the training data limited.
- Future work will include validation of detection performance predictions with the HLA for assumed known signal wavefronts using MFP techniques.
- Evaluation of theory and prediction in the presence of interference and environmental uncertainty is planned using the SWELLEX-96 S59 event data.

Duke University School of Engineering

Incorporating Environmental Uncertainty Into Bayesian Sonar Detection Performance Prediction



Detection performance prediction
ROC (probability of detection vs probability of false alarm)

Environmental uncertainty

- Uncertain channel parameters ψ
  - Water depth, sound speed profile, etc
- Unknown signal source position S_s
- Unknown interference source position S_k

Probability density functions $p(\psi)$, $p(S_s)$, and $p(S_k)$ capture uncertainties in the environmental parameters

Duke University School of Engineering

From Environmental Uncertainty to Wave Front Uncertainty

$$H_1 \quad r = \sqrt{SNR(\psi, S_s)}\, a s(\psi, S_s) + n_1, \quad n_1 = \sum_{k=1}^{K} \sqrt{INR_k(\psi, S_k)}\, b f_k(\psi, S_k) + n_0$$

$$H_0 \quad r = n_1, \qquad a \sim N(0,1), b \sim N(0,1), n_0 \sim N(0, I_N)$$

- Environmental uncertainty characterized by the probability density functions

$$\psi \sim p(\psi), S_s \sim p(S_s), S_k \sim p(S_k)$$

- Uncertain signal wave front $s(\psi, S_s) = \mathbf{H}(\psi, S_s) / \left\| \mathbf{H}(\psi, S_s) \right\|$
- Ocean transfer function sampled at an N sensor array $\mathbf{H}(\psi, S_s)$
- Signal Matrix: matrix of signal wave fronts due to environmental uncertainty

$$\Im = [s_1, s_2, \ldots s_L] = [s((\Psi, S_s)_1), s((\Psi, S_s)_2), \ldots s((\Psi, S_s)_L)]$$

- Total signal-to-noise ratio at receivers $SNR(\psi, S_s) = \dfrac{\sigma_a^2}{\sigma_n^2} \mathbf{H}(\psi, S_s)^H \mathbf{H}(\psi, S_s)$

Duke University School of Engineering

From Wave Front Uncertainty to Bayesian Sonar Performance Prediction

- Discrete likelihood ratio: Monte Carlo integration over uncertainty

$$\lambda(r) = \frac{1}{L}\sum_{i=1}^{L}\lambda_i, \quad \lambda_i = \frac{1}{1+SNR_i}\exp\left(\frac{SNR_i\left|r^H s_i\right|^2}{1+SNR_i}\right)$$

$$SNR_i = SNR((\Psi, S_s)_i)$$

- Detection performance: Monte Carlo over data to get $p(\lambda|H_1)$ and $p(\lambda|H_0)$

$$P_D = \int_{\beta}^{\infty} d\lambda p(\lambda \mid H_1) \qquad P_F = \int_{\beta}^{\infty} d\lambda p(\lambda \mid H_0)$$

- Disadvantage of Monte Carlo performance evaluation methods
  - Lack of insight into fundamental parameters
  - Computationally intensive

- Motivates developing analytical Bayesian sonar performance predictions

Duke University School of Engineering

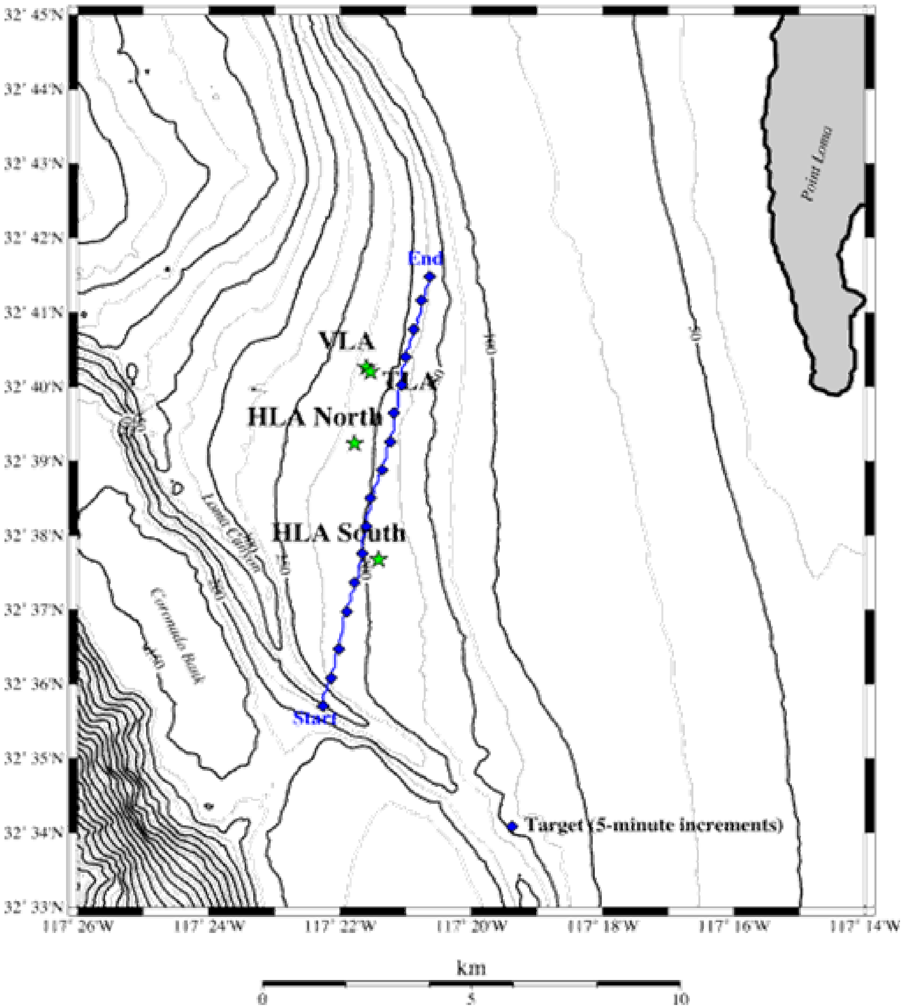
Analytical Bayesian Sonar Performance Prediction

Problem		Detector statistic	ROC performance
Known Ocean s: s(ψ,S_s) f: f(ψ,S_k)	Multiple Interferers	$\lambda'(r) = \left\| r^H M_0^{-1} s \right\|^2$ $M_0 = \sum_{k=1}^{K} INR_k f_k f_k^H + I_N$	$P_D = P_F^{1/(SNR\, s^H M_0^{-1} s+1)}$
	Single Interferer	$\lambda'(r) = \left\| r^H (I - \frac{INR}{1+INR} ff^H) s \right\|^2$	$P_D = P_F^{1/(SNR\,(1-\frac{INR}{1+INR}\left\|s^H f\right\|^2)+1)}$
	Diffuse noise	$\lambda'(r) = \left\| r^H s \right\|^2$	$P_D = P_F^{1/(SNR+1)}$
Uncertain Ocean s: s(ψ,S_s) R: rank of the signal matrix	Multiple Interferers	$\lambda(r) = \frac{1}{L}\sum_{i=1}^{L} \lambda_i$ $\lambda_i = \frac{1}{1+SNR_i A_i} \exp(\frac{SNR_i \left\| r^H M_0^{-1} s_i \right\|^2}{1+SNR_i A_i})$ $SNR_i = SNR((\Psi, S_s)_i), \quad A_i = s_i^H M_0^{-1} s_i$ $M_0 = \frac{1}{M}\sum_{m=1}^{M}\sum_{k=1}^{K} INR_k \quad ((\Psi, S_k)_m)$ $f_k((\Psi, S_k)_m) f_k((\Psi, S_k)_m)^H + I_N$	$P_D = 1-(1-P_F)^{(R-1)/R}$ $(1-(1-(1-P_F)^{1/R})^{1/(SNR\,A+1)})$ $SNR = E_{\Psi,S_s}(SNR(\Psi, S_s))$ $A = E_{\Psi,S_s}(s(\Psi,S_s)^H M_0^{-1} s(\Psi,S_s))$
	Diffuse noise	$\lambda(r) = \frac{1}{L}\sum_{i=1}^{L} \lambda_i$ $\lambda_i = \frac{1}{1+SNR_i} \exp(\frac{SNR_i \left\| r^H s_i \right\|^2}{1+SNR_i})$	$P_D = 1-(1-P_F)^{(R-1)/R}$ $(1-(1-(1-P_F)^{1/R})^{1/(SNR+1)})$

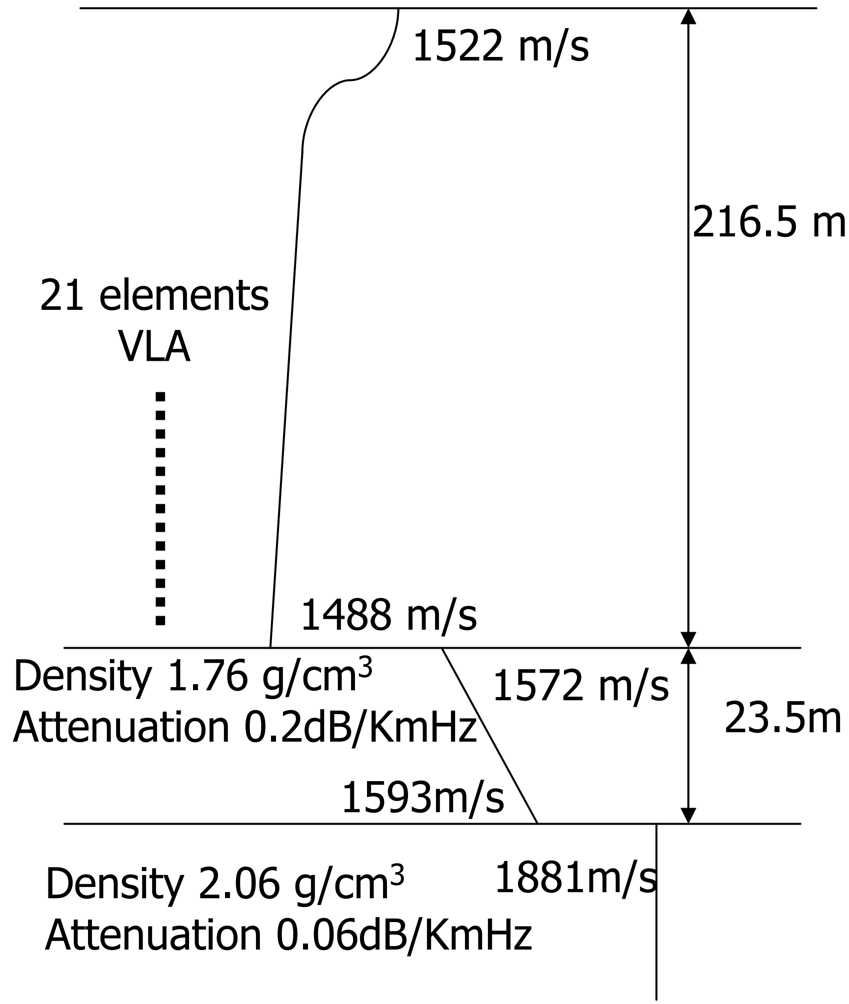
SWellEx-96 Event S5

5/10/96 23:15-5/11/96 00:30 GMT

Environmental Model

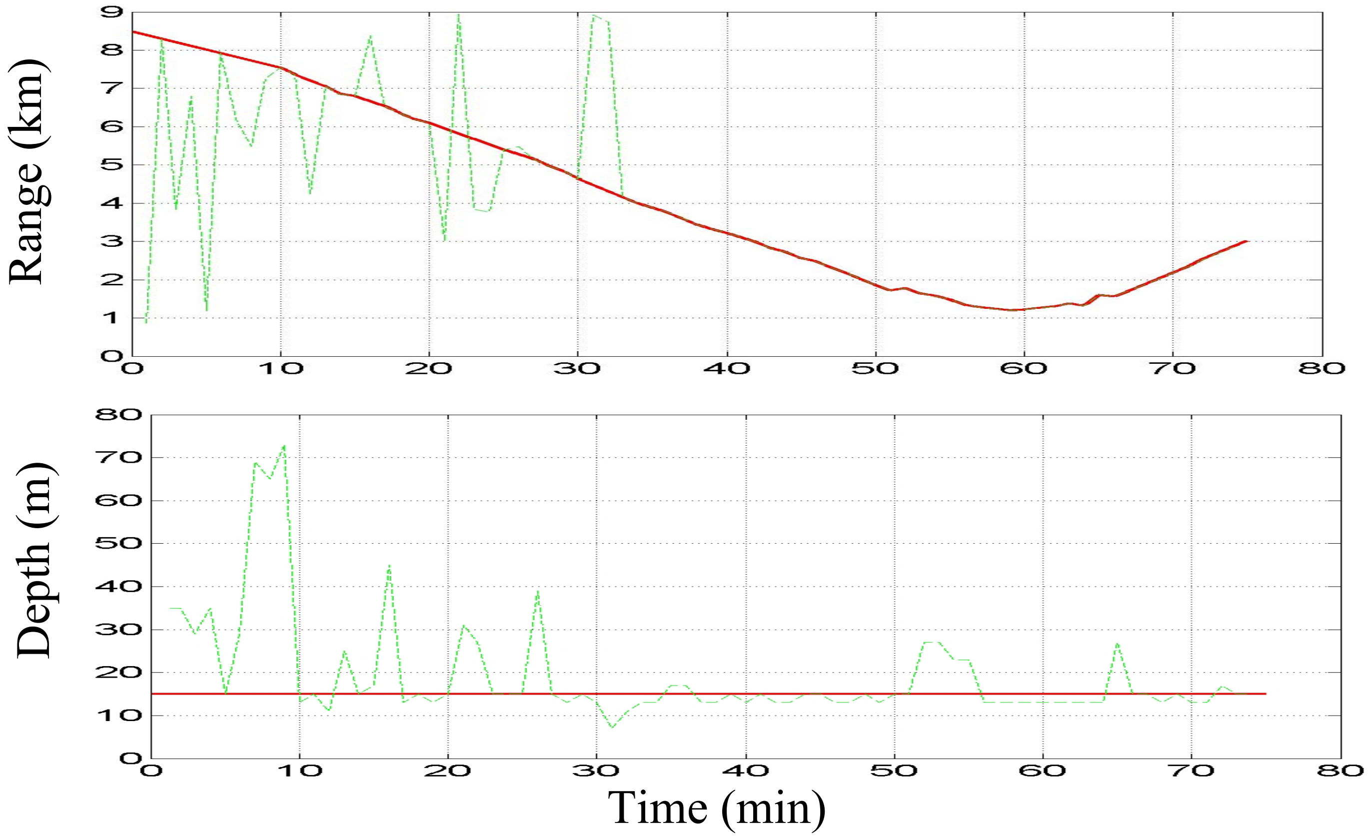


Courtesy of MPL/SIO



Duke University
School of Engineering

Source Track SWellEx-96 Event S5



- Used for replica field computation in detection problems
- Estimated using Bartlett processor with nominal environmental parameters, 109Hz data

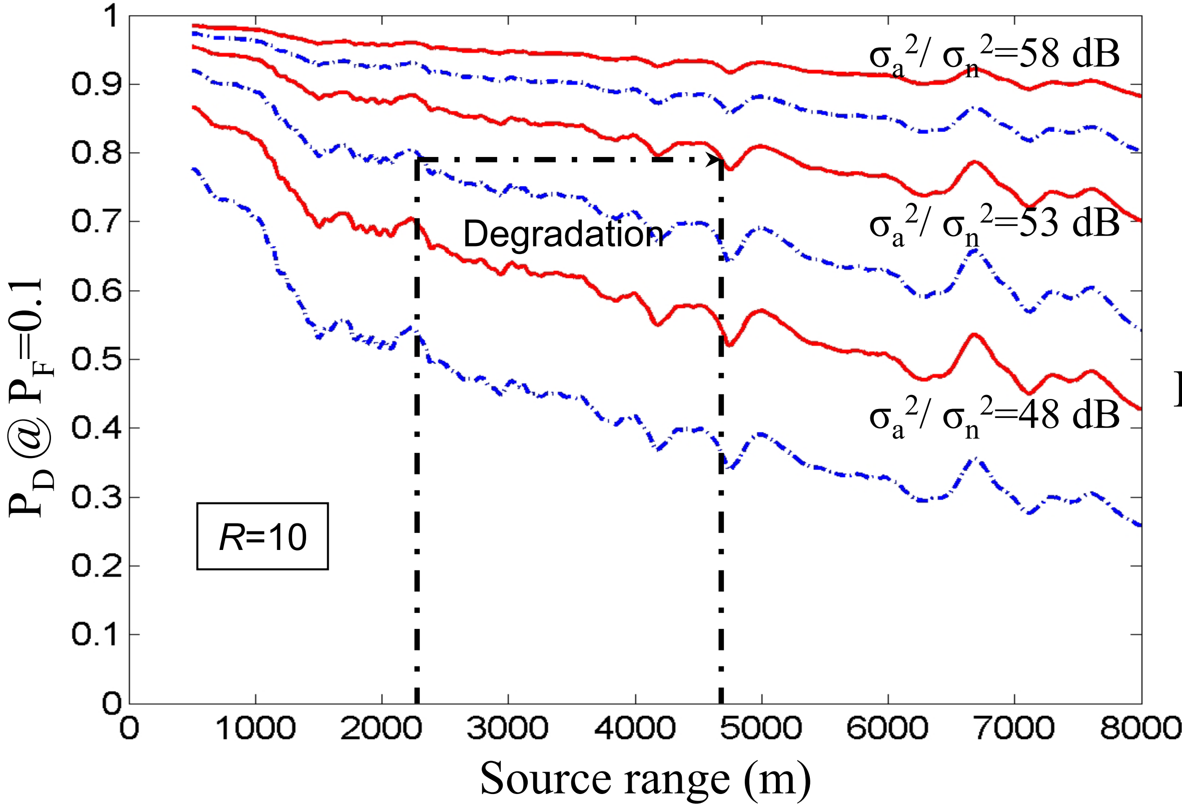
Detection Performance Prediction

An Example: SWellEx-96 Event S5 Environment

Matched $P_D = P_F^{1/(SNR+1)}$

Bayesian $P_D = 1-(1-P_F)^{(R-1)/R}(1-(1-(1-P_F)^{1/R})^{1/(SNR+1)})$

$SNR = \sigma_a^2 \mathbf{H}(\psi, S_s)^H \mathbf{H}(\psi, S_s)/\sigma_n^2$



σ_a^2 Signal variance

σ_n^2 Noise variance

ψ Ocean parameters

S_s Source position parameters

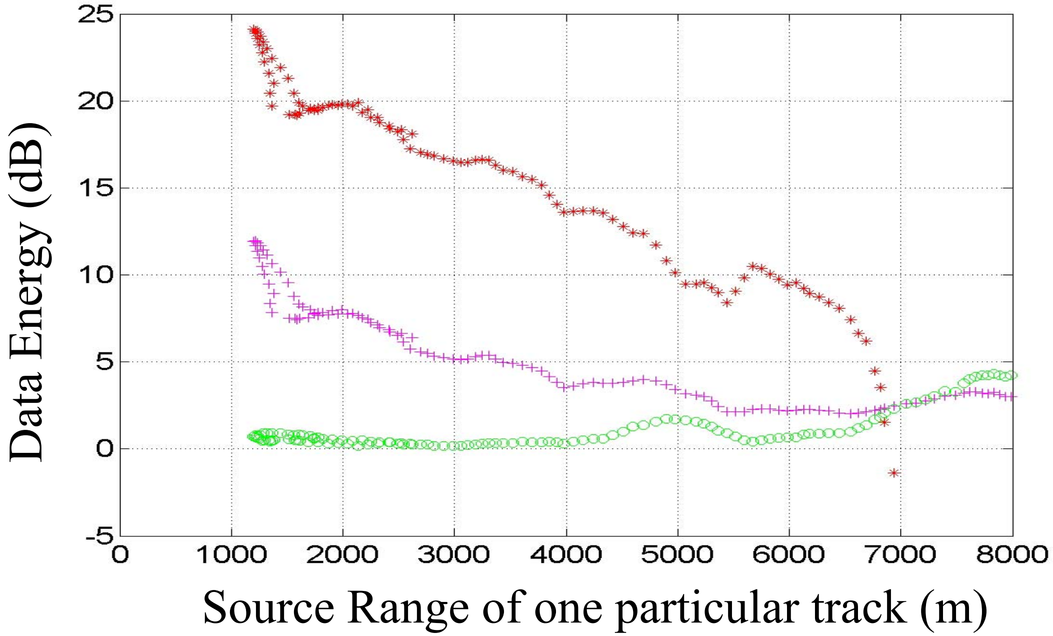
$\mathbf{H}(\psi, S_s)$ Replica field computed using KRAKEN code

R Rank of the signal matrix

Duke University School of Engineering

SNR Estimation

Experimental Results (SWellEx-96 Event S5)



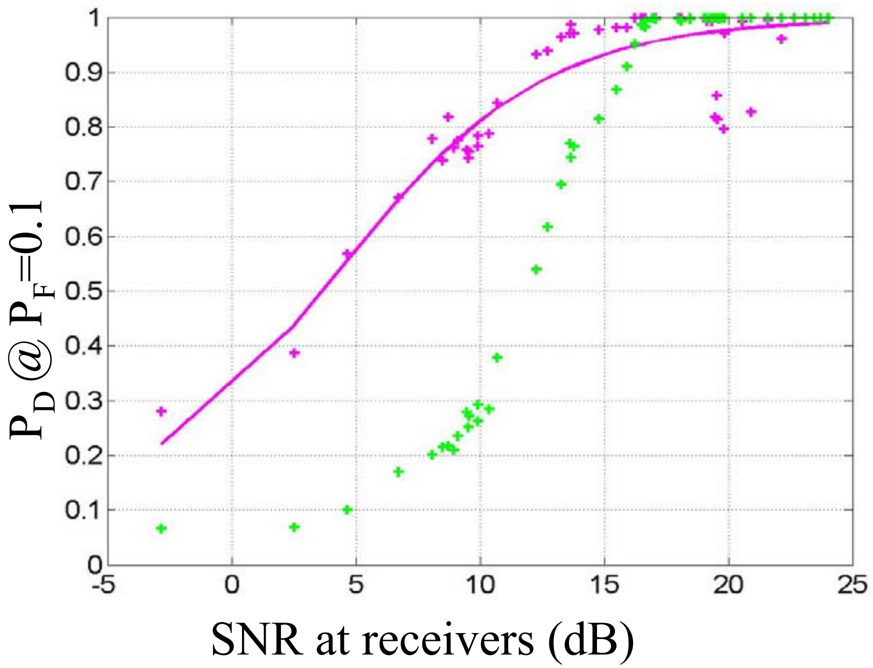
- **+** SNR estimation = channels*$(\|r_1\|^2 - \|r_0\|^2)/\|r_0\|^2$
- **+** $\|r_1\|^2$: r_1 data energy in each frame
- **o** $\|r_0\|^2$: r_0 data energy in each frame

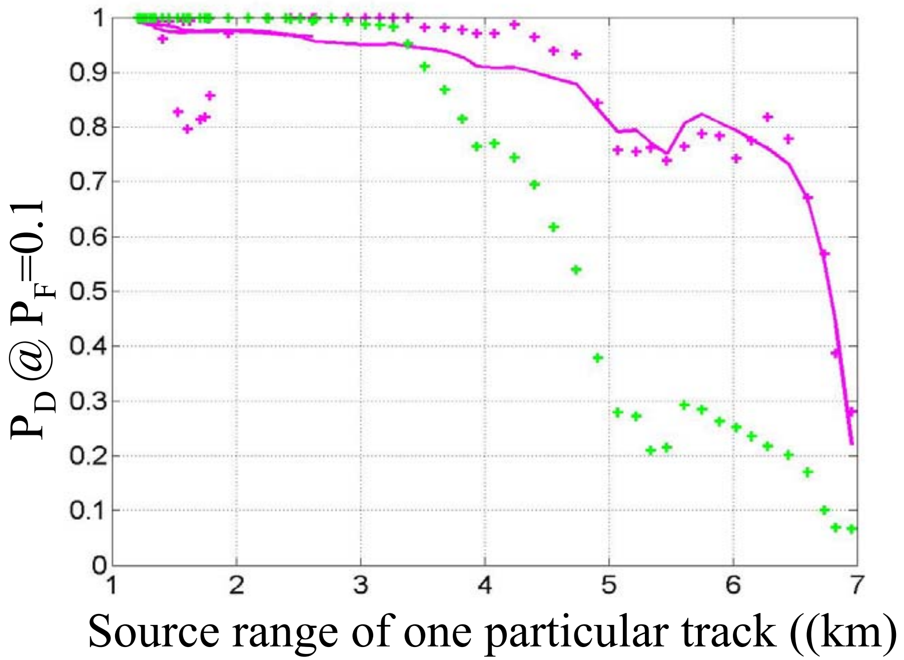
H_1: r_1 = r109+(0.88-1) r99, H_0: r_0= 0.88 r99

Frame: 21 channels x 900snapshots (5mins), updated per 100snapshots

Detection Performance Prediction

Experimental Results (SWellEx-96 Event S5) **: Diffuse Noise**





+ Matched-ocean detector $\lambda'(r) = \left| r^H s \right|^2$

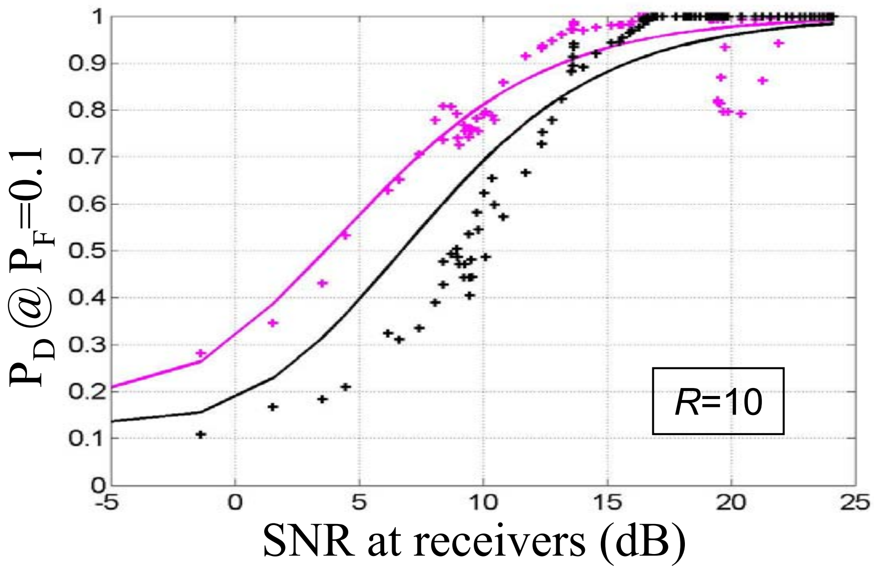
+ Energy detector $\lambda'(r) = r^H r$

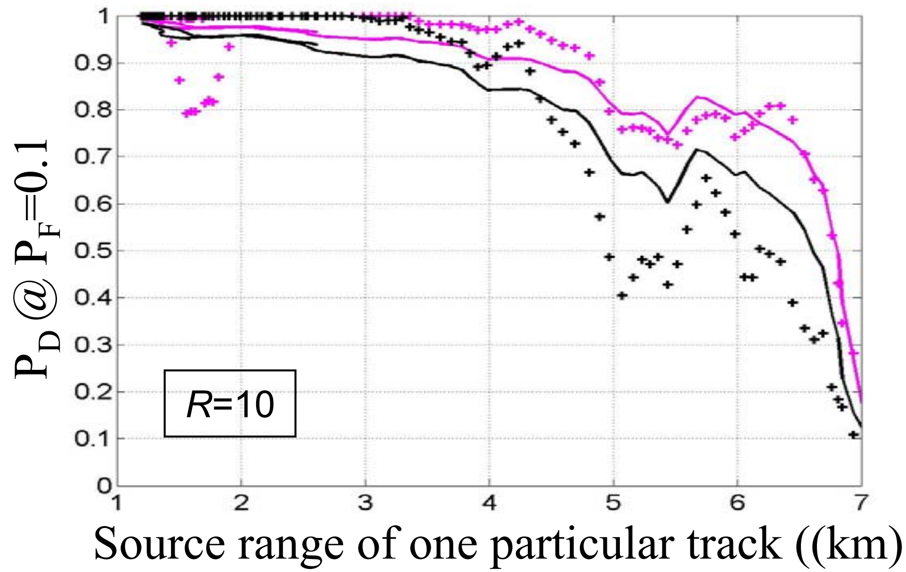
– Analytical performance prediction $P_D = P_F^{1/(SNR+1)}$

Detection Performance Prediction

Experimental Results (SWellEx-96 Event S5) :

Wave Front Uncertainty Due to Source Motion





+ Matched-ocean detector

$$\lambda = \left| r^H s \right|^2$$

— Analytical performance prediction

$$P_D = P_F^{1/(SNR+1)}$$

+ Bayesian detector

$$\lambda(r) = \frac{1}{L} \sum_{i=1}^{L} \lambda_i$$

$$\lambda_i = \frac{1}{1+SNR_i} \exp\left(\frac{SNR_i \left| r^H s_i \right|^2}{1+SNR_i}\right)$$

— Analytical performance prediction

$$P_D = 1-(1-P_F)^{(R-1)/R}(1-(1-(1-P_F)^{1/R})^{1/(SNR+1)})$$

Duke University School of Engineering

Summary and Future Work

- Derived fast, analytical methods for characterizing the performance of optimum Bayesian and adaptive CFAR detectors.
- Derived signal covariance matrix rank as an efficient measure of ocean environmental uncertainty for Bayesian detection performance prediction.
- Evaluated expected detection loss due to ocean environmental uncertainty, as a function of SNR and signal covariance matrix rank, for vertical and horizontal arrays in representative environments.
- Evaluated expected detection loss due to limited noise covariance training data, in the presence of signal wavefront uncertainty, for adaptive CFAR detectors.
- Demonstrated good agreement between analytical detection performance predictions and the performance of optimal detectors, matched to the degree of environmental uncertainty, using SWELLEX-96 S5 event data.
- Future work will include the comparison of detection performance predictions in the presence of interference and environmental uncertainty using the SWELLEX-96 S59 event data.
- Evaluation of comparative performance prediction accuracy using horizontal versus vertical array configurations in an uncertain environment are also planned.

Duke University School of Engineering